

AREAS RELATED TO CIRCLES

PPT-6

SUBJECT : MATHEMATICS

CHAPTER NUMBER: 12

CHAPTER NAME : AREAS RELATED TO CIRCLES

CHANGING YOUR TOMORROW

PREVIOUS KNOWLEDGE TEST

Length of an Arc and Area of Sector

(i) The length of an arc of a sector of an angle θ is given by, $\frac{2\pi R\theta}{360^\circ}$

The area of the sector $A = \frac{\theta}{360^\circ} \times \pi r^2$ where θ is given by,

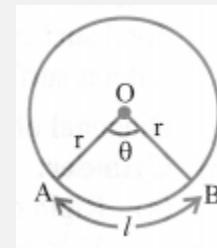
Area of Segment

i) Area of segment APB = Area (sector OAPB) – Area(Δ OAB)

This is the area of minor segment.

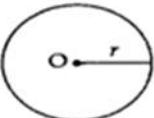
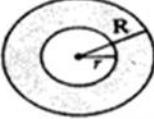
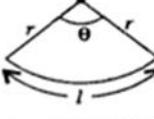
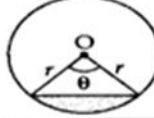
\therefore area of major segment AQB = πr^2 – Area of minor segment APB

(ii) If θ is the central angle, then the area of segment APB



$$= \frac{\theta}{360^\circ} \times \pi r^2 - r^2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}$$

TABLE FOR AREA AND PERIMETER

Figures	Area	Perimeter	
Circle 	πr^2 or $\frac{\pi d^2}{4}$	$2\pi r$ or πd	r : radius d : diameter $\pi = \frac{22}{7}$ or 3.14
Semicircle 	$\frac{\pi r^2}{2}$	$\pi r + 2r$	
Quadrant 	$\frac{\pi r^2}{4}$	$\frac{\pi r}{2} + 2r$	
Ring 	$\pi(R + r)(R - r)$	$2\pi R$ (Outer circumference) $2\pi r$ (Inner circumference)	R : Radius of bigger circle r : Radius of smaller circle
Sector 	(i) $\frac{\theta}{360} \times \pi r^2$ (ii) $\frac{1}{2} lr$	$\frac{\theta}{360} \times 2\pi r + 2r$	r : Radius of circle l : length of arc
Segment 	$\frac{\theta}{360} \pi r^2 - \frac{1}{2} r^2 \sin \theta$	$\frac{\pi r \theta}{180} + 2r \sin \frac{\theta}{2}$	θ : angle subtended by arc at centre

LEARNING OUTCOME

- 1 .Students will be able to find the area of combined plane figures.
- 2.Students will be able to identify angle subtended by the sector at the Centre.
3. Students will be able to apply the knowledge of Area of sector and segment of a circle in solving real life problems..

Area of combined plane figures

https://youtu.be/vrXCfSg_-PO {10.12}

1. Find the area of a quadrant of a circle, where the circumference of circle is 44 cm. (Use $\pi = 22/7$)

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Circumference of a circle = 44 cm

$$\Rightarrow 2\pi r = 44 \text{ cm}$$

$$\Rightarrow 2 \times \frac{22}{7} \times r = 44 \text{ cm}$$

$$\Rightarrow r = \frac{44 \times 7}{2 \times 22} = 7 \text{ cm}$$

$$\text{Area of a quadrant} = \frac{1}{4} \pi r^2 = \frac{1}{4} \times \frac{22}{7} \times 7 \times 7$$

$$\therefore \text{Area of quadrant} = \frac{77}{2} = 38.5 \text{ cm}^2$$

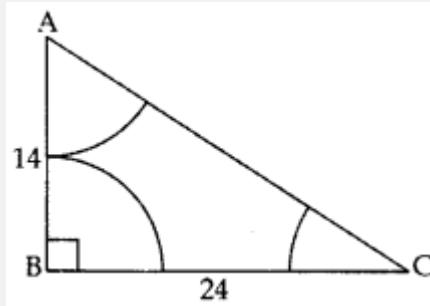
2. Area of a sector of a circle of radius 14 cm is 154 cm^2 . Find the length of the corresponding arc of the sector.

2. Area of a sector of a circle of radius 14 cm is 154 cm². Find the length of the corresponding arc of the sector.

Area of sector = 154 cm²

$$\begin{aligned}\frac{1}{2}lr &= 154 & \Rightarrow & \quad \frac{1}{2}(l)(14) = 154 \\ \Rightarrow 7l &= 154 & \Rightarrow & \quad l = 22 \text{ cm} \\ \therefore \text{Length of the corresponding arc, } l &= 22 \text{ cm}\end{aligned}$$

3. ABC is a triangle right-angled at B, with AB = 14 cm and BC = 24 cm. With the vertices A, B and C as centres, arcs are drawn, each of radius 7 cm. Find the area of the shaded region. (Use $\pi = 22/7$)



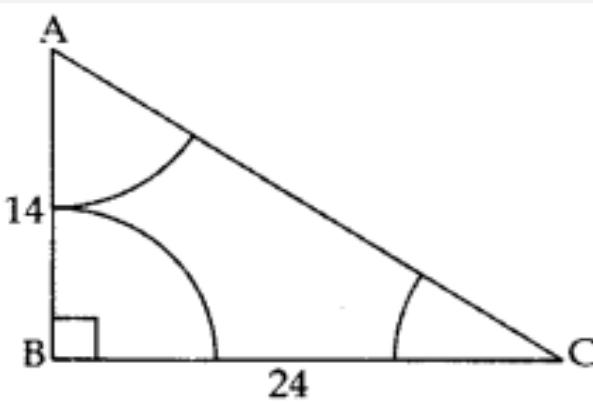
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Let $\angle BAC = \theta_1$, $\angle ABC = \theta_2$ and $\angle ACB = \theta_3$

Area of the shaded region

$$= \text{ar}(\Delta ABC) - [\text{ar}(\text{sector A}) + \text{ar}(\text{sector B}) + \text{ar}(\text{sector C})]$$

$$\begin{aligned}
 &= \frac{1}{2} \times AB \times BC - \left[\frac{\theta_1}{360} \pi r^2 + \frac{\theta_2}{360} \pi r^2 + \frac{\theta_3}{360} \pi r^2 \right] \\
 &= \frac{1}{2} \times 14 \times 24 - \frac{\pi r^2}{360} (\theta_1 + \theta_2 + \theta_3) \\
 &= 168 - \frac{1}{360} \times \frac{22}{7} \times 7 \times 7 \times 180 \\
 &= 168 - 77 = 91 \text{ cm}^2 \quad \dots [\because \theta_1 + \theta_2 + \theta_3 = 180]
 \end{aligned}$$



4. All the vertices of a rhombus lie on a circle. Find the area of the rhombus, if the area of the circle is 1256 cm^2 . [Use $\pi = 3.14$]

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Let r be the radius of circle

In rhombus, $AB = BC = CD = AD$

$$\Rightarrow AC = BD = 2r$$

$$\therefore \text{Area of circle} = \pi r^2$$

$$1256 = 3.14r^2$$

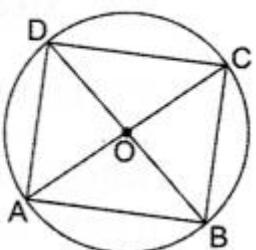
...[Given]

$$r^2 = \frac{1256}{3.14} = 400$$

$$\Rightarrow r = 20 \text{ cm}$$

$$\therefore \text{Area of rhombus} = \frac{1}{2} (AC \times BD)$$

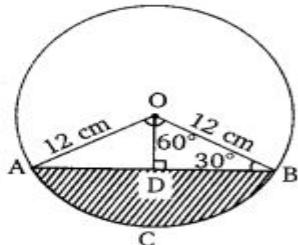
$$= \frac{1}{2} \times 40 \times 40 = 800 \text{ cm}^2$$



5. A chord of a circle of the radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)..

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Let AB be a chord which subtends an angle 120° at the centre O of the circle.



Area of segment ACB

$$= \text{Area of sector OACB} - \text{Area of } \triangle OAB \quad \dots(i)$$

$$\text{Area of sector OACB} = \frac{120^\circ}{360^\circ} \times 3.14 \times (12)^2$$

$$= \frac{1}{3} \times 3.14 \times 12 \times 12 = 150.72 \text{ cm}^2. \quad \dots(ii)$$

We draw $OD \perp AB$.

$$\therefore \angle OBD = 180^\circ - (90^\circ + 60^\circ) = 30^\circ$$

$$\text{Now from } \triangle ODB, \sin 30^\circ = \frac{OD}{OB}$$

$$\Rightarrow \frac{1}{2} = \frac{OD}{12} \quad \left[\because \sin 30^\circ = \frac{1}{2} \right]$$

$$\Rightarrow OD = 6 \text{ cm}$$

$$\text{Also, } \cos 30^\circ = \frac{BD}{OB} \Rightarrow \frac{\sqrt{3}}{2} = \frac{BD}{12} = \frac{12\sqrt{3}}{2} \text{ cm}$$

$$\Rightarrow BD = 6\sqrt{3} \text{ cm}$$

$$\therefore AB = 2BD = 2 \times 6\sqrt{3} \text{ cm} = 12\sqrt{3} \text{ cm.}$$

$$\therefore \text{Area of } \triangle OAB = \frac{1}{2} \times AB \times OD$$

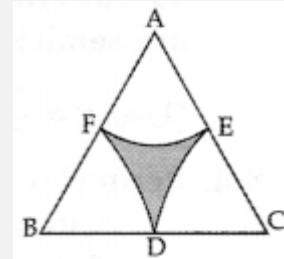
$$= \frac{1}{2} \times 12\sqrt{3} \times 6 = 36\sqrt{3} \text{ cm}^2. \quad \dots(iii)$$

From equations (i), (ii) and (iii), we get:

$$\begin{aligned} \text{Area of segment ACB} &= (150.72 - 36\sqrt{3}) \text{ cm}^2 \\ &= 88.37 \text{ cm}^2 \end{aligned}$$

Hence, the area of the segment of the circle is **88.37 cm²**.

6. Arcs are drawn by taking vertices A, B and C of an equilateral triangle ABC of side 14 cm as centres to intersect the sides BC, CA and AB at BZ their respective mid-points D, E and F. Find the area of the shaded region. [Use $\pi = 22/7$ and $\sqrt{3} = 1.73$]



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$\angle ABC = \angle BAC = \angle ACB = 60^\circ$... [equilateral Δ

$$\text{Let } \theta = 60^\circ, \quad r = \frac{14}{2} = 7 \text{ cm}$$

Area of shaded region

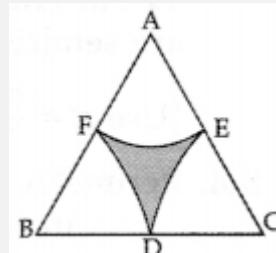
$$= \text{ar}(\Delta ABC) - 3 \text{ (ar of sector)}$$

$$= \frac{\sqrt{3}}{4} (\text{side})^2 - 3 \cdot \frac{\theta}{360} \pi r^2$$

$$\text{...[Area of equilateral } \Delta = \frac{\sqrt{3}}{4} \text{ side}^2$$

$$= \frac{1.73}{4} \times 14 \times 14 - 3 \times \frac{60}{360} \times \frac{22}{7} \times 7 \times 7$$

$$= 84.77 - 77 = 7.77 \text{ cm}^2$$



HOME ASSIGNMENT:CH-12



EXERCISE-12.1 ,12.2 AND 12.3



THANKING YOU
ODM EDUCATIONAL GROUP