PERIOD 3

MATHEMATICS

CHAPTER NUMBER :~ 8
CHAPTER NAME :~QUADRILATERALS

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PREVIOUS KNOWLEDGE TEST

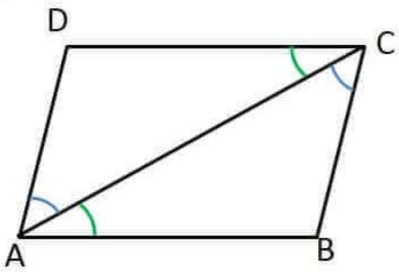
1. Prove that each angle of a rectangle is 90°.

LEARNING OUTCOME:~

1.Students will be able to learn more properties of parallelogram.
2.Students will be able to solve sums based on parallelogram.

Theorem 8.4:

In a parallelogram, opposite angles are equal



Given: A parallelogram ABCD

with AC as its diagonal

To prove: ∠ A = ∠ C & ∠ B = ∠ D

Proof:

Opposite sides of parallelogram is parallel

So, AB ∥ DC and AD ∥ BC

Since **AB ∥ DC**

& AC is the transversal

∠ BAC = ∠ DCA *(Alternate angles)* ...(1)

Since **AD ∥ BC**

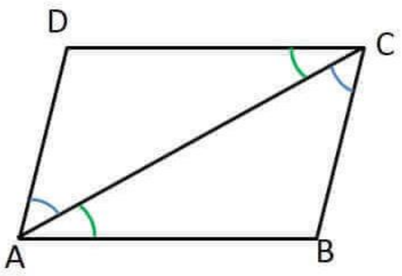
& AC is the transversal

∠ DAC = ∠ BCA *(Alternate angles)* ...(2)

Adding (1) and (2)

$\angle BAC + \angle DAC = \angle DCA + \angle BCA$

$\angle BAD = \angle DCB$



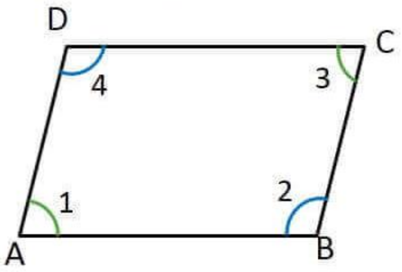
Similarly,

we can prove $\angle ADC = \angle ABC$

Hence Proved.

Theorem 8.5

If in a quadrilateral, each pair of opposite angles is equal, then it is a parallelogram.



Given : ABCD is a quadrilateral with opposite angles equal,

i.e. $\angle 1 = \angle 3$ & $\angle 2 = \angle 4$

To Prove : ABCD is a Parallelogram

Proof : By angle sum property of quadrilateral

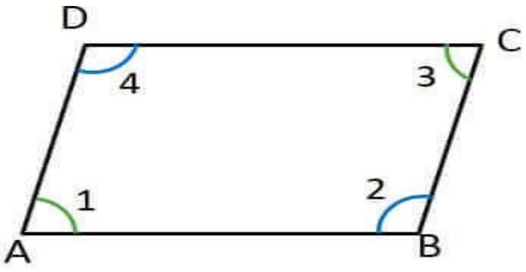
$$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360^\circ$$

$$\angle 1 + \angle 2 + \angle 1 + \angle 2 = 360^\circ \quad \text{(Given } \angle 1 = \angle 3 \text{ \& } \angle 2 = \angle 4\text{)}$$

$$2(\angle 1 + \angle 2) = 360^\circ$$

$$\angle 1 + \angle 2 = \frac{360°}{2}$$

$$\angle 1 + \angle 2 = 180° \quad ...(1)$$



Similarly we can prove that,

$$\angle 1 + \angle 4 = 180° \quad ...(2)$$

For lines **AD and BC**

with AB as transversal,

$\angle 1$ and $\angle 2$ are interior angles on the same side of transversal, and

$$\angle 1 + \angle 2 = 180°$$

Since interior angles on same side of transversal are supplementary,

Hence, AD || BC

For lines **AB and DC**

with AD as transversal,

$\angle 1$ and $\angle 4$ are interior angles on the same side of transversal, and

$$\angle 1 + \angle 4 = 180°$$

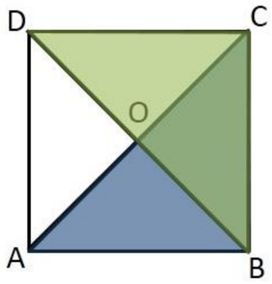
Since interior angles on same sid

of transversal are supplementary

Hence, AB || DC

Ex 8.1, 4

Show that the diagonals of a square are equal and bisect each other at right angles.



Given: ABCD be a square.

Diagonals intersect at O.

To prove : We need to prove 3 things

1. The diagonals of a square are equal ,i.e. AC = BD
2. bisect each other, i.e. OA = OC & OB = OD,
3. at right angles ,any of ∠AOB , ∠BOC , ∠COD , ∠AOD is 90°

Proof:

In ΔABC and ΔDCB,

AB = DC *(Sides of square are equal)*

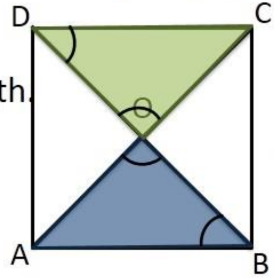
∠ABC = ∠DCB *(Both 90° , as all angles of square are 90°)*

BC = BC *(Common)*

∴ ΔABC ≅ ΔDCB *(SAS congruence rule)*

⇒ AC = DB (CPCT) ...(1)

Hence, the diagonals of a square are equal in length.



Now we need to prove diagonals bisect each other i.e. AO = CO, BO = DO

In ΔAOB and ΔCOD,

∠AOB = ∠COD (Vertically opposite angles)

∠ABO = ∠CDO *(AB ∥ CD & BD as transversal, alternate angles equal)*

AB = CD (Sides of square are equal)

∴ ΔAOB ≅ ΔCOD (AAS congruence rule)

∴ AO = CO and OB = OD (CPCT) ...(2)

Hence, the diagonals of a square bisect each other.

In ΔAOB and ΔCOB,

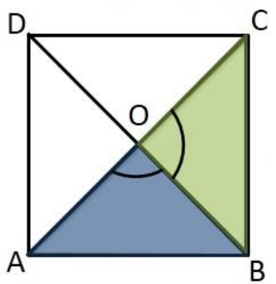
OA = OC (From (2))

AB = BC (Sides of square are equal)

BO = BO (Common)

∴ ΔAOB ≅ ΔCOB (SSS congruency)

∴ ∠AOB = ∠COB (CPCT) ...(3)



Now

∠AOB + ∠COB = 180° (Linear Pair)

∠AOB + ∠AOB = 180° (From (3))

2∠AOB = 180°

$\angle AOB = \frac{180°}{2}$

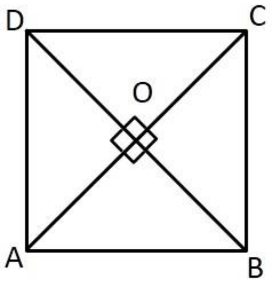
∠AOB = 90°

Hence, AC & BD bisect at right angles

Hence proved

Ex 8.1, 5

Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.



Given:

Let ABCD be the quadrilateral.

Diagonals are equal, i.e., AC = BD ...(1)

& bisect each other, i.e. OA = OC & OB = OD, ...(2)

at right angles ,i.e., $\angle AOB = \angle BOC = \angle COD = \angle AOD = 90°$ ...(3)

To prove: ABCD is a square

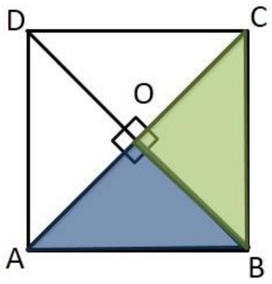
Proof: Square is a parallelogram with all sides equal and one angle 90°

First we will prove ABCD is a parallelogram

and then prove all sides equal , and one angle equal to 90°

In ΔAOB and ΔCOB,

OA = OC	(From (2))
∠AOB = ∠COB	(From (3), both 90°)
OB = OB	(Common)
∴ ΔAOB ≅ ΔCOB	(SAS congruence rule)
∴ AB = CB	(CPCT)



Similarly we can prove

ΔAOB ≅ ΔDOA , so AB = AD

& ΔBOC ≅ ΔCOD , so CB = DC

So, AB = AD = CB = DC

Now we can say that

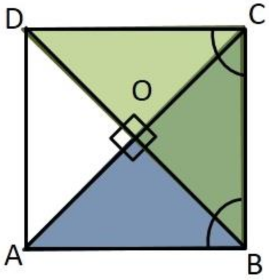
AB = CD & AD = BC

In ABCD, both pairs of opposite sides are equal,

Hence, ABCD is a parallelogram

Square is a parallelogram with all sides equal and one angle 90°

So, we prove one angle 90°



In ΔABC and ΔDCB,

AC = BD (From (1))

AB = DC (Opposite sides of parallelogram are equal)

BC = CB (Common)

∴ ΔABC ≅ ΔDCB (SSS congruence rule)

⇒∠ ABC = ∠ DCB (CPCT) ...(4)

Now,

AB ∥ CD (Opposite sides of parallelogram are parallel)

& BC is transversal

∠ B + ∠ C = 180° *(Interior angles on same side of transversal is supplementary)*

∠ B + ∠ B = 180° (From (4))

$2\angle B = 180°$

$\angle B = \frac{180°}{2} = 90°$

Thus, ABCD is a parallelogram with all sides equal and one angle 90°

So, ABCD is a square

HOMEWORK ASSIGNMENT

Exercise 8.1
Question number 4,5

AHA

1.Show that the diagonals of a rhombus bisect each other at right angles.

THANKING YOU

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