

MODEL LESSON

MATHEMATICS

CHAPTER NUMBER :~ 2

CHAPTER NAME :~ POLYNOMIALS

SUB TOPIC :~ FACTOR THEOREM AND REMAINDER THEOREM

CHANGING YOUR TOMORROW

PREVIOUS KNOWLEDGE TEST

Factorize $6x^2 + 17x + 5$ by splitting the middle term and by using factor theorem.

LEARNING OUTCOME:~

Students will learn

- a) factor theorem and remainder theorem.

EXERCISE~2.4

Question 1.

Determine which of the following polynomials has $(x + 1)$ a factor.

- (i) $x^3 + x^2 + x + 1$
- (ii) $x^4 + x^3 + x^2 + x + 1$
- (iii) $x^4 + 3x^3 + 3x^2 + x + 1$
- (iv) $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

Solution:

The zero of $x + 1$ is -1 .

(i) Let $p(x) = x^3 + x^2 + x + 1$
 $\therefore p(-1) = (-1)^3 + (-1)^2 + (-1) + 1$.
 $= -1 + 1 - 1 + 1$
 $\Rightarrow p(-1) = 0$
So, $(x + 1)$ is a factor of $x^3 + x^2 + x + 1$.

(ii) Let $p(x) = x^4 + x^3 + x^2 + x + 1$

$$\therefore p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$$

$$= 1 - 1 + 1 - 1 + 1$$

$$\Rightarrow p(-1) \neq 1$$

So, $(x + 1)$ is not a factor of $x^4 + x^3 + x^2 + x + 1$.

(iii) Let $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$.

$$\therefore p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$$

$$= 1 - 3 + 3 - 1 + 1 = 1$$

$$\Rightarrow p(-1) \neq 0$$

So, $(x + 1)$ is not a factor of $x^4 + 3x^3 + 3x^2 + x + 1$.

(iv) Let $p(x) = x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

$$\therefore p(-1) = (-1)^3 - (-1)^2 - (2 + \sqrt{2})(-1) + \sqrt{2}$$

$$= -1 - 1 + 2 + \sqrt{2} + \sqrt{2}$$

$$= 2\sqrt{2}$$

$$\Rightarrow p(-1) \neq 0$$

So, $(x + 1)$ is not a factor of $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$.

Question 2.

Use the Factor Theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases

- (i) $p(x) = 2x^3 + x^2 - 2x - 1, g(x) = x + 1$
- (ii) $p(x) = x^3 + 3x^2 + 3x + 1, g(x) = x + 2$
- (iii) $p(x) = x^3 - 4x^2 + x + 6, g(x) = x - 3$

Solution:

(i) We have, $p(x) = 2x^3 + x^2 - 2x - 1$ and $g(x) = x + 1$

$$\therefore p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$$

$$= 2(-1) + 1 + 2 - 1$$

$$= -2 + 1 + 2 - 1 = 0$$

$\Rightarrow p(-1) = 0$, so $g(x)$ is a factor of $p(x)$.

(ii) We have, $p(x) = x^3 + 3x^2 + 3x + 1$ and $g(x) = x + 2$

$$\therefore p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1$$

$$= -8 + 12 - 6 + 1$$

$$= -14 + 13$$

$$= -1$$

$\Rightarrow p(-2) \neq 0$, so $g(x)$ is not a factor of $p(x)$.

(iii) We have, $p(x) = x^3 - 4x^2 + x + 6$ and $g(x) = x - 3$

$$\therefore p(3) = (3)^3 - 4(3)^2 + 3 + 6$$

$$= 27 - 4(9) + 3 + 6$$

$$= 27 - 36 + 3 + 6 = 0$$

$\Rightarrow p(3) = 0$, so $g(x)$ is a factor of $p(x)$.

Question 3.

Find the value of k , if $x - 1$ is a factor of $p(x)$ in each of the following cases

(i) $p(x) = x^2 + x + k$

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$

(iv) $p(x) = kx^2 - 3x + k$

Solution:

For $(x - 1)$ to be a factor of $p(x)$, $p(1)$ should be equal to 0.

(i) Here, $p(x) = x^2 + x + k$

Since, $p(1) = (1)^2 + 1 + k$

$\Rightarrow p(1) = k + 2 = 0$

$\Rightarrow k = -2.$

(ii) Here, $p(x) = 2x^2 + kx + \sqrt{2}$

Since, $p(1) = 2(1)^2 + k(1) + \sqrt{2}$

$$= 2 + k + \sqrt{2} = 0$$

$$k = -2 - \sqrt{2} = -(2 + \sqrt{2})$$

(iii) Here, $p(x) = kx^2 - \sqrt{2}x + 1$

Since, $p(1) = k(1)^2 - (1) + 1$

$$= k - \sqrt{2} + 1 = 0$$

$$\Rightarrow k = \sqrt{2} - 1$$

(iv) Here, $p(x) = kx^2 - 3x + k$

$$p(1) = k(1)^2 - 3(1) + k$$

$$= k - 3 + k$$

$$= 2k - 3 = 0$$

$$\Rightarrow k = 34$$

<https://www.youtube.com/watch?v=MOYifLWM~4M>

“As great a genius as Archimedes could not invent analytical geometry, for the algebraic knowledge necessary for such as achievement was not available in his time...”

~Nathan A. Court...

Factorization of Polynomials

Factor Theorem- If $p(x)$ is a polynomial of degree n where n is greater than or equal to one and a is any real number then-

- 1) $x - a$ is a factor of $p(x)$ if $p(a) = 0$
- 2) $p(a) = 0$ if $x - a$ is a factor of $p(x)$

Proof-

Using Remainder theorem, we can write-

$$p(x) = [(x - a) \times q(x)] + p(a) \quad \text{Here, } r(x) = r = p(a)$$

- (1) Suppose, $p(a) = 0$. Then $p(x) = (x - a) \times q(x)$

Clearly, $x - a$ is a factor of $p(x)$

- (2) Suppose $x - a$ is a factor of $p(x)$. Then we can write, $p(x) = (x - a) \times g(x)$ for some polynomial $g(x)$. So, $p(a) = 0$

Applications of Factor Theorem

Example 1. Find the value of k , if $x - 1$ is a factor of $4x^3 + 3x^2 - 4x + k$

Solution: By Factor theorem, $p(1) = 0$

$$\begin{aligned} p(1) &= 4(1) + 3(1) - 4(1) + k = 0 \\ K &= -3 \end{aligned}$$

Factorization of Quadratic polynomials using Splitting the middle term method

Consider the quadratic polynomial $a x^2 + b x + c$

Suppose $(p x + q)$ and $(r x + s)$ be its factors.

So we can write, $a x^2 + b x + c = (p x + q) \times (r x + s)$

On comparing the coefficients of the like terms in L.H.S and R.H.S we get,

$a = p r$, $b = p s + q r$, $c = q s$

And clearly, $(p s) \times (q r) = (p r) \times (q s) = (a c)$.

So, to factorize the quadratic polynomial, we need to write b as the sum of two numbers whose product is $(a c)$

For example- Factorize $6x^2 + 17x + 5$ by-

- 1) Splitting the middle term method
- 2) Factor theorem

Solution-

- 1) We can write $6x^2 + 17x + 5 = 6x^2 + (15 + 2)x + 5$
 $= 6x^2 + 15x + 2x + 5$
 $= 3x(2x + 5) + 1(2x + 5)$
 $= (3x + 1)(2x + 5)$

- 2) We can write $6x^2 + 17x + 5 = 6(x^2 + (17/6)x + (5/6))$

Taking, $p(x) = x^2 + (17/6)x + (5/6)$

So, $6x^2 + 17x + 5 = 6p(x)$

Suppose a and b are zeroes of $p(x)$. Then $x - a$ and $x - b$ are factors of $p(x)$ and we have

$$p(x) = (x - a)(x - b)$$

So, $6x^2 + 17x + 5 = 6(x - a)(x - b) \longrightarrow (*)$

On comparing the coefficients of like terms, we get, $a + b = 5/6$

Possibilities for the values of a and b satisfying $a + b = 5/6$ are- $\frac{\pm 1}{2}, \frac{\pm 1}{3}, \frac{\pm 5}{3}, \frac{\pm 5}{2}, \pm 1$

Also, these values of a and b must satisfy $p(a) = 0$ and $p(b) = 0$

So, by checking all the values found above, we get that $a = -1/3$ and $b = -5/2$ satisfies the above conditions. So, $(*)$, becomes- $6x^2 + 17x + 5 = (3x + 1)(2x + 5)$

To factorize a cubic polynomial $p(x)$, we first find a zero of the polynomial by hit and trial method (by putting different values of x in the polynomial equation $p(x) = 0$)

Then the obtained root of the polynomial equation say $x = a$ (or simply, the zero of the polynomial $p(x)$) implies that $x - a$ is a factor of $p(x)$.

So we can write, $p(x) = (x - a) g(x) \longrightarrow (**)$

where $g(x)$ is a polynomial such that the degree of $g(x) <$ degree of $p(x)$

Here, as degree of $p(x) = 3$ So, degree of $g(x) = 2$ i.e. $g(x)$ is a quadratic polynomial and using factor theorem or splitting the middle term method we can factorize $g(x)$ and thus we get the desired factorization by putting the resultant $g(x)$ in (**)

Evaluation:-

1. Factorize: $-y^2 - 5y + 6$.
2. Factorize: $2x^2 + 7x + 3$.

HOMEWORK:-
EXERCISE - 2.4
QUESTION NUMBER-4,5

AHA:~

1. If $x^2 - 1$ is a factor of $ax^4 + bx^3 + cx^2 + dx + e$

Show that $a+c+e = b+d = 0$.

2. Without actual division , prove that $2x^4 - 5x^3 + 2x^2 - x + 2$ is actually divisible by $x^2 - 3x + 2$.

THANKING YOU
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