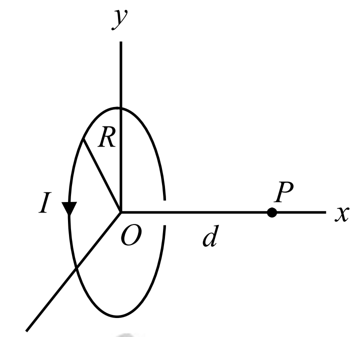
Chapter- 5

MAGNETISM AND MATTER

CURRENT LOOP AS A MAGNETIC DIPOLE AND ITS MAGNETIC MOMENT

The magnitude of magnetic field on the central axis (x-axis) at a point at a distance of d from the centre O of the circular coil of radius R having N number of turns carrying current I (fig-1) is



$[Fig-1]$

$$B = \frac{\mu_0}{4\pi}\frac{2NIA}{\left(R^2+d^2\right)^{3/2}} \text{---eq.(1)}$$

and its direction is along the x-axis. Here μ_0 is the magnetic permeability of a vacuum.

If $d \gg R$ then eq.(1) becomes

$$B \approx \frac{\mu_0}{4\pi}\frac{2NIA}{d^3} \text{---eq.(2)}$$

We define the magnetic moment of the current loop as,

$$\vec{\mathbf{m}} = I\vec{\mathbf{A}}$$

where the direction of the area vector $\vec{\mathbf{A}}$ is given by the right-hand thumb rule and is directed along the positive x-axis in fig. (1).

So in vector form eq.(2) is

$\vec{\mathbf{B}} \approx \dfrac{\mu_0}{4\pi}\dfrac{2N\vec{\mathbf{m}}}{d^3}$ ---eq.(3)

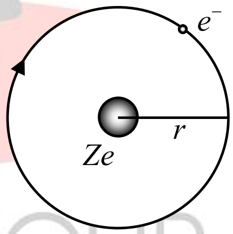
TASK FOR THE STUDENT-

Compare above eq.(3) with electric field due to an ideal electric dipole at an end-on point.

UNIT AND DIMENSION OF MAGNETIC DIPOLE MOMENT- SI unit is $\mathrm{A.m^2}$. Dimension is $\mathrm{AL^2}$ or $\mathrm{IL^2}$.

MAGNETIC DIPOLE MOMENT OF REVOLVING ELECTRON

The electron of charge (–e) performs uniform circular motion around a stationary heavy nucleus of charge +Ze. This constitutes a current I, given by



$[Fig.(2)]$

$I = \dfrac{e}{T}$ ---(1)

If T is the time period of revolution, r be the orbital radius, and v the orbital speed of the electron then,

$v = \dfrac{2\pi r}{T}$ ---(2)

So eq.(1) becomes

$I = e\frac{v}{2\pi r}$ ---eq.(3)

There will be a magnetic moment, associated with this circulating current given by,

$\mu = I\pi r^2$

$= e\frac{v}{2\pi r}\pi r^2$

$= \frac{evr}{2}$

$= \frac{emvr}{2m}$

$= \frac{el}{2m}$ ---eq.(4)

Bohr hypothesised that the angular momentum assumes a discrete set of values, namely,

$l = \frac{nh}{2\pi}$ ---eq.(5)

where n is a natural number, n = 1, 2, 3, ... and h is Planck's constant. named after Max Planck constant) with a value $h = 6.626\times10^{-34}\,\mathrm{J\cdot s}$. So,

$\mu = \frac{e}{2m}\frac{nh}{2\pi}$

$= n\frac{eh}{4\pi m}$

$= n\mu_B$ ---eq.(6)

where,

$\mu_B = \frac{eh}{4\pi m}$ ---eq.(7)

is called Bohr magneton and its value is $\mu_B = 9.27 \times 10^{-24}\,\mathrm{A \cdot m^2}$

In fig. (2) the negatively charged electron is moving clockwise, leading to an anticlockwise current. From the right-hand rule, the magnetic moment is perpendicular to the plane of the paper and outward. The angular momentum of the electron w.r.t. the nucleus is perpendicular to the plane of the paper and inward. So in vector form

$\vec{\mathbf{\mu}} = -\frac{e\vec{\mathbf{l}}}{2m}$ ---eq.(8)

The ratio

$\frac{\mu}{l} = \frac{e}{2m}$ ---eq.(9)

is called the gyromagnetic ratio and is a constant. Its value is $8.8 \times 10^{10}\,\mathrm{C/kg}$ for an electron.

TASK FOR STUDENT-

Show that $\mu_B = 9.27 \times 10^{-24}\,\mathrm{A \cdot m^2}$ and the gyromagnetic ratio is $8.8 \times 10^{10}\,\mathrm{C/kg}$ for an electron.

PROBLEM-

Obtain the frequency of revolution of the electron in its circular orbit. Does the answer depend on the speed of the electron? Explain. (NCERT EXERCISE)

PROBLEM-

The magnetic moment vectors μ_S and μ_l associated with the intrinsic spin angular momentum S and orbital angular momentum l, respectively, of an electron are predicted by quantum theory (and verified experimentally to a high accuracy) to be given by:

$$\vec{\boldsymbol{\mu}}_S = -\frac{e}{m}\vec{\mathbf{S}},$$

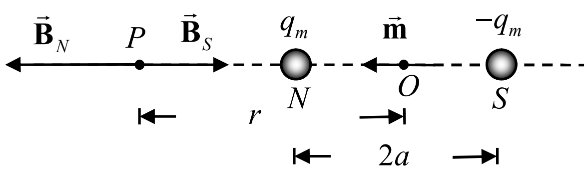
$$\vec{\boldsymbol{\mu}}_l = -\frac{e}{2m}\vec{\mathbf{l}}$$

Which of these relations is in accordance with the result expected classically? Outline the derivation of the classical result. (NCERT EXERCISE)

MAGNETIC FIELD INTENSITY DUE TO A MAGNETIC DIPOLE (BAR MAGNET) ALONG ITS AXIS

Magnetic charge (or pole strength) of the north pole and the south poles are respectively q_m and $-q_m$. The magnetic field intensities at P due to north and south poles are respectively

$$\vec{\mathbf{B}}_N = \frac{\mu_0}{4\pi}\frac{q_m}{(r-a)^2}\hat{\mathbf{m}}$$



[fig.(1)]

$$\vec{\mathbf{B}}_S = \frac{\mu_0}{4\pi}\frac{-q_m}{(r+a)^2}\hat{\mathbf{m}}$$

where $\hat{\mathbf{m}}$ is the unit vector along the dipole axis (from south pole to north pole)?

So the total field at P is

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_N + \vec{\mathbf{B}}_S$$

$$= \frac{\mu_0}{4\pi} q_m \left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\hat{\mathbf{m}}$$

$$= \frac{\mu_0}{4\pi}\frac{4raq_m}{(r^2-a^2)^2}\hat{\mathbf{m}}$$

$$= \frac{\mu_0}{4\pi}\frac{2r\vec{\mathbf{m}}}{(r^2-a^2)^2}$$

where, $\vec{\mathbf{m}} = 2aq_m\hat{\mathbf{m}}$ is the magnetic moment of a bar magnet.

For $r \gg a$,

$$\vec{\mathbf{B}} \approx \frac{\mu_0}{4\pi}\frac{2\vec{\mathbf{m}}}{r^3}$$

QUESTION

Two identical looking iron bars A and B are given, one of which is definitely known to be magnetised. (We do not know which one.) How would one ascertain whether or not both are magnetised? If only one is magnetised, how does one ascertain which one? [Use nothing else but the bars A and B.] (NCERT)

ANS-

Try to bring different ends of the bars closer. A repulsive force in some situation establishes that both are magnetised. If it is always attractive, then one of them is not magnetised. In a bar magnet, the intensity of the magnetic field is the strongest at the two ends (poles) and weakest at the central region. This fact may be used to determine whether A or B is the magnet. In this case, to see which one of the two bars is a magnet, pick up one, (say, A) and lower one of its ends; first on one of the ends of the other (say, B), and then on the middle of B. If you notice that in the middle of B, A experiences no force, then B is magnetised. If you do not notice any change from the end to the middle of B, then A is magnetised.

QUESTION

What happens if a bar magnet is cut into two pieces: (i) transverse its length, (ii) along its length? (NCERT)

ANS-

In either case, one gets two magnets, each with a north and south pole.

PROBLEM

What is the magnitude of the axial field due to a bar magnet of length 5.0 cm at a distance of 50 cm from its mid-point? The magnetic moment of the bar magnet is 0.40 $\mathrm{A \cdot m^2}$. (NCERT)

SOL-

$$B \approx \frac{\mu_0}{4\pi}\frac{2m}{r^3} = 6.4\times10^{-7}\,\mathrm{T}$$

PROBLEM

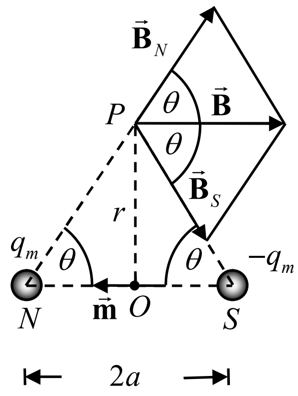
Two magnetic poles (north and south) ±10 $\mu\mathrm{A \cdot m}$ are placed 5.0 mm apart. Determine the magnetic field at a point on the axis of the dipole 15 cm away from its centre on the side of the north pole.

PROBLEM

A bar magnet has pole strengths ±10 $\mu\mathrm{A \cdot m}$ located at points N: (0, 0, –15 cm) and S: (0,0, +15 cm), respectively. What is the magnetic dipole moment of the system?

MAGNETIC FIELD INTENSITY DUE TO A MAGNETIC DIPOLE (BAR MAGNET) ALONG PERPENDICULAR TO ITS AXIS

Magnetic charge (or pole strength) of the north pole and the south poles are respectively q_m and $-q_m$. The magnitudes of magnetic field intensities at P due to north and south poles are respectively



[fig.(1)]

$$B_N = \frac{\mu_0}{4\pi}\frac{q_m}{r^2+a^2}$$

$$B_S = \frac{\mu_0}{4\pi}\frac{q_m}{r^2+a^2}$$

and are equal.

The directions of $\vec{\mathbf{B}}_N$ and $\vec{\mathbf{B}}_S$ are as shown in fig. (1). The components normal to the dipole axis cancel away. The components along the dipole axis add up. The total magnetic field is opposite to the unit vector $\hat{\mathbf{m}}$ along the dipole axis from south pole to north pole). We have

So the total field at P is

$$\vec{\mathbf{B}} = -\left(B_N + B_S\right)\cos\theta\hat{\mathbf{m}}$$

$$= -\frac{\mu_0}{4\pi}\frac{2aq_m}{\left(r^2+a^2\right)^{3/2}}\hat{\mathbf{m}}$$

$$= -\frac{\mu_0}{4\pi}\frac{\vec{\mathbf{m}}}{\left(r^2+a^2\right)^{3/2}}$$

where, $\vec{\mathbf{m}} = 2aq_m\hat{\mathbf{m}}$ is the magnetic moment of a bar magnet.

For $r \gg a$,

$$\vec{\mathbf{B}} \approx -\frac{\mu_0}{4\pi}\frac{\vec{\mathbf{m}}}{r^3}$$

PROBLEM

What is the magnitude of the equatorial field due to a bar magnet of length 5.0 cm at a distance of 50 cm from its mid-point? The magnetic moment of the bar magnet is 0.40 $\mathrm{A \cdot m^2}$. (NCERT)

SOL-

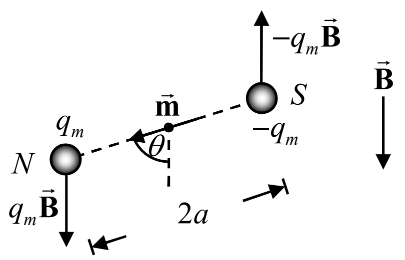
$$B \approx \frac{\mu_0}{4\pi}\frac{m}{r^3} = 3.2\times10^{-7}\,\mathrm{T}$$

PROBLEM

Two magnetic poles (north and south) ±10 $\mu\mathrm{A \cdot m}$ are placed 5.0 mm apart. Determine the magnetic field at a point 15 cm away from its centre on a line passing through the centre and normal to the axis of the dipole.

TORQUE ON A MAGNETIC DIPOLE IN A UNIFORM MAGNETIC FIELD

Consider a magnetic dipole of the dipole moment $\vec{\mathbf{m}}$ in a uniform external magnetic field $\vec{\mathbf{B}}$, as shown in fig. (1).



[fig.(1)]

There is a force $q_m\vec{\mathbf{B}}$ on the north pole and a force $-q_m\vec{\mathbf{B}}$ on the south pole. The net force on the dipole is zero since $\vec{\mathbf{B}}$ is uniform. These forces act at different points, resulting in a torque on the dipole. When the net force is zero, the torque (couple) is independent of the origin. Its magnitude equals the magnitude of each force multiplied by the arm of the couple (perpendicular

distance between the two antiparallel forces).

The magnitude of torque = $q_m B 2a \sin\theta = mB\sin\theta$

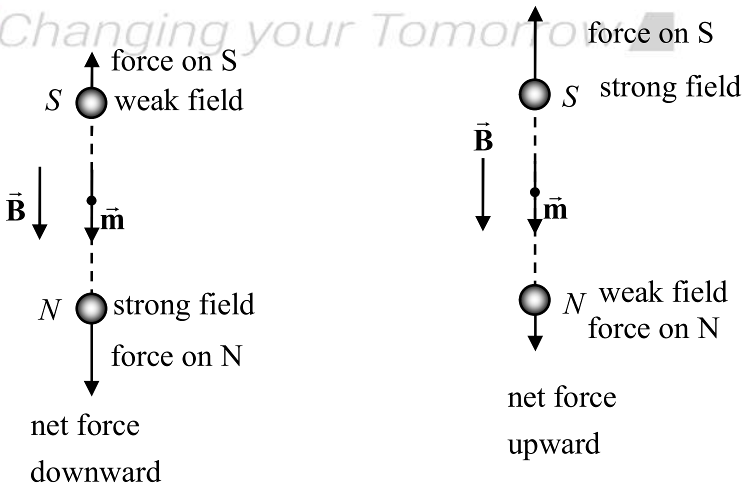
Its direction is normal to the plane of the paper, coming out of it.

The magnitude of $\vec{\mathbf{m}}\times\vec{\mathbf{B}}$ is also $mB\sin\theta$ and its direction is normal to the paper, coming out of it. Thus,

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{m}}\times\vec{\mathbf{B}}$$

This torque will tend to align the dipole with the field $\vec{\mathbf{B}}$.

When $\vec{\mathbf{m}}$ is aligned with $\vec{\mathbf{B}}$, the torque is zero. In the non-uniform field, the net force will evidently be non-zero. In addition, there will, in general,



[fig.(2)][dipole in non-uniform field]

be a torque on the system as before. The general case is involved, so let us consider the simpler situations when $\vec{\mathbf{m}}$ is parallel to $\vec{\mathbf{B}}$ or antiparallel to $\vec{\mathbf{B}}$. In either case, the net torque is zero, but there is a net force on the dipole if $\vec{\mathbf{B}}$ is not uniform.

We may at times be required to determine the magnitude of $\vec{\mathbf{B}}$ accurately. This is done by placing a small compass needle of known magnetic moment $\vec{\mathbf{m}}$ and moment of inertia I and allowing it to oscillate in the magnetic field. The equation of motion of the dipole is

$$I\frac{d^2\theta}{dt^2} = -mB\sin\theta$$

$$\Rightarrow \frac{d^2\theta}{dt^2} + \frac{mB}{I}\sin\theta = 0$$

For small values of θ in radians, the above equation becomes

$$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0 \text{ (angular SHM)}$$

The angular frequency and time period are respectively

$$\omega = \sqrt{\frac{mB}{I}}$$

$$T = 2\pi\sqrt{\frac{I}{mB}}$$

$$\Rightarrow B = \frac{4\pi^2 I}{mT^2}$$

The magnetic potential energy U_m is given by

$$U_m = \int \tau d\theta$$

$$= \int mB \sin\theta d\theta$$

$$= -mB\cos\theta$$

$$= -\vec{\mathbf{m}} \cdot \vec{\mathbf{B}}$$

Here zero of potential energy is at θ= 90º, i.e., when the needle is perpendicular to the field. Equation (5.6) shows that potential energy is minimum (= –mB) at θ= 0º (most stable position) and maximum (= +mB) at θ= 180º (most unstable position).

PROBLEM

The magnetic needle has a magnetic moment $6.7\times10^{-2}\,\mathrm{A\cdot m^2}$ and moment of inertia $I = 7.5\times10^{-6}\,\mathrm{kg\cdot m^2}$. It performs 10 complete oscillations in 6.70 s. What is the magnitude of the magnetic field? (NCERT)

SOL-

$$B = \frac{4\pi^2 I}{mT^2} = \frac{4\pi^2\left(7.5\times10^{-6}\,\mathrm{kg\cdot m^2}\right)}{\left(6.7\times10^{-2}\,\mathrm{A\cdot m^2}\right)\left(0.67\mathrm{s}\right)^2} = 0.01\mathrm{T}$$

PROBLEM

A short bar magnet placed with its axis at 30º with an external field of 800 G experiences a torque of $0.016\text{N}\cdot\text{m}$. (a) What is the magnetic moment of the magnet? (b) What is the work done in moving it from its most stable to most unstable position? (NCERT)

SOL-

$$\tau = mB\sin\theta$$

$$\Rightarrow m = 0.4\text{A}\cdot\text{m}^2$$

$$W = -mB\cos 180° + mB\cos 0° = 2mB$$

$$\Rightarrow W = 0.064\text{J}$$

QUESTION-

(a) A magnetised needle in a uniform magnetic field experiences a B torque but no net force. An iron nail near a bar magnet, however, experiences a force of attraction in addition to torque. Why? (b) Must every magnetic configuration have a north pole and a south pole? What about the field due to a toroid? (NCERT)

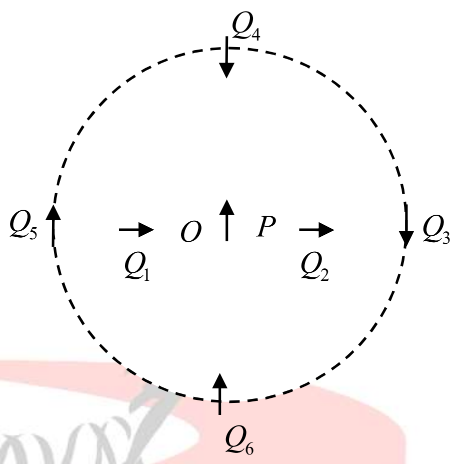
ANS-

(a)No force if the field is uniform. The iron nail experiences a non-uniform field due to the bar magnet. There is induced magnetic moment in the nail, therefore, it experiences both force and torque.

The net force is attractive because the induced south pole (say) in the nail is closer to the north pole of a magnet than the induced north pole.

(b) Not necessarily. True only if the source of the field has a net nonzero magnetic moment. This is not so for a toroid or even for a straight infinite conductor.

QUESTION-



[fig.(3)]

Figure 3 shows a small magnetised needle P placed at a point O. The arrow shows the direction of its magnetic moment. The other arrows show different positions (and orientations of the magnetic moment) of another identical magnetised needle Q. (a) In which configuration the system is not in equilibrium? (b) In which configuration is the system in (i) stable, and (ii) unstable equilibrium? (c) Which configuration corresponds to the lowest potential energy among all the configurations shown? (NCERT)

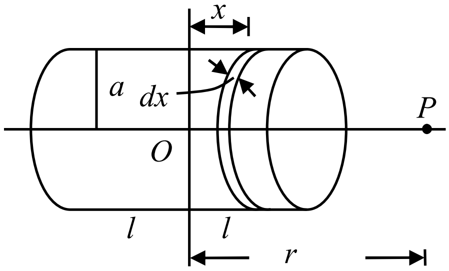
ANS-

(a) PQ_1 and PQ_2

(b) (i) PQ_3, PQ_6 (stable); (ii) PQ_5, PQ_4 (unstable)

(c) PQ_6

BAR MAGNET AS AN EQUIVALENT SOLENOID



[fig.(1)]

Let the solenoid of fig. (1) consists of n turns per unit length. Let its length be 2l and radius a. We can evaluate the axial field at a point P, at a distance r from the centre O of the solenoid. To do this, consider a circular element of thickness dx of the solenoid at a distance x from its centre. It consists of ndx turns. Let I be the current in the solenoid. The magnitude of the field at point P due to the circular element is

$$dB = \frac{\mu_0}{4\pi}\frac{2ndxI\pi a^2}{\left\{a^2+(r-x)^2\right\}^{3/2}}$$

The magnitude of the total field at P is

$$B = 2\frac{\mu_0}{4\pi}\int_{x=-l}^{l}\frac{ndxI\pi a^2}{\left\{a^2+(r-x)^2\right\}^{3/2}}$$

Let $r \gg a$ and $r \gg l$. So

$$B = 2\frac{\mu_0}{4\pi}\int_{x=-l}^{l}\frac{ndxI\pi a^2}{r^3}$$

$$\Rightarrow B = 2\frac{\mu_0}{4\pi}\frac{nI\pi a^2}{r^3}\int_{x=-l}^{l}dx$$

$$\Rightarrow B = 2\frac{\mu_0}{4\pi}\frac{nI\pi a^2}{r^3}2l$$

$$\Rightarrow B = \frac{\mu_0}{4\pi}\frac{2m}{r^3}$$

This is also the far axial magnetic field of a bar magnet which one may obtain experimentally. Thus, a bar magnet and a solenoid produce similar magnetic fields. The magnetic moment of a bar magnet is thus equal to the magnetic moment of an equivalent solenoid that produces the same magnetic field.

PROBLEM

A solenoid of cross-sectional area $2\times10^{-4}\,\mathrm{m}^2$ and 1000 turns, has a magnetic moment $0.4\ \mathrm{A}\cdot\mathrm{m}^2$. Determine the current flowing through the solenoid. (NCERT)

SOL-

$$I = \frac{m}{NA} = \frac{0.4\ \mathrm{A}\cdot\mathrm{m}^2}{1000\times2\times10^{-4}\,\mathrm{m}^2} = 2\mathrm{A}$$

QUESTION

Must every magnetic configuration have a north pole and a south pole? What about the field due to a toroid? (NCERT)

ANS-

Not necessarily. True only if the source of the field has a net nonzero magnetic moment. This is not so for a toroid or even for a straight infinite conductor.

PROBLEM

A closely wound solenoid of 800 turns and area of cross-section $2.5\times10^{-4}\,m^2$ carries a current of 3.0 A. Explain the sense in which the solenoid acts like a bar magnet. What is its associated magnetic moment? If the solenoid is free to turn about the vertical direction and a uniform horizontal magnetic field of 0.25 T is applied, what is the magnitude of the torque on the solenoid when its axis makes an angle of 30° with the direction of applied field? (NCERT)

PROPERTIES OF MAGNETIC FILED LINES

(i) The magnetic field lines of a magnet (or a solenoid) form continuous closed loops. This is unlike the electric dipole where these field lines begin from a positive charge and end on the negative charge or escape to infinity.

(ii) The tangent to the field line at a given point represents the direction of the net magnetic field B at that point.

(iii) The larger the number of field lines crossing per unit area, the stronger is the magnitude of the magnetic field B.

(iv) The magnetic field lines do not intersect, for if they did, the direction of the magnetic field would not be unique at the point of intersection.

QUESTION-

(a)Magnetic field lines show the direction (at every point) along which a small magnetised needle aligns (at the point). Do the magnetic field lines also represent the lines of force on a moving charged particle at every point?

(b) Magnetic field lines can be entirely confined within the core of a toroid, but not within a straight solenoid. Why?

(c) Does a bar magnet exert a torque on itself due to its field? Does one element of a current-carrying wire exert a force on another element of the same wire?

(d) Magnetic field arises due to charges in motion. Can a system have magnetic moments even though its net charge is zero? (NCERT)

ANS-

(a)No,

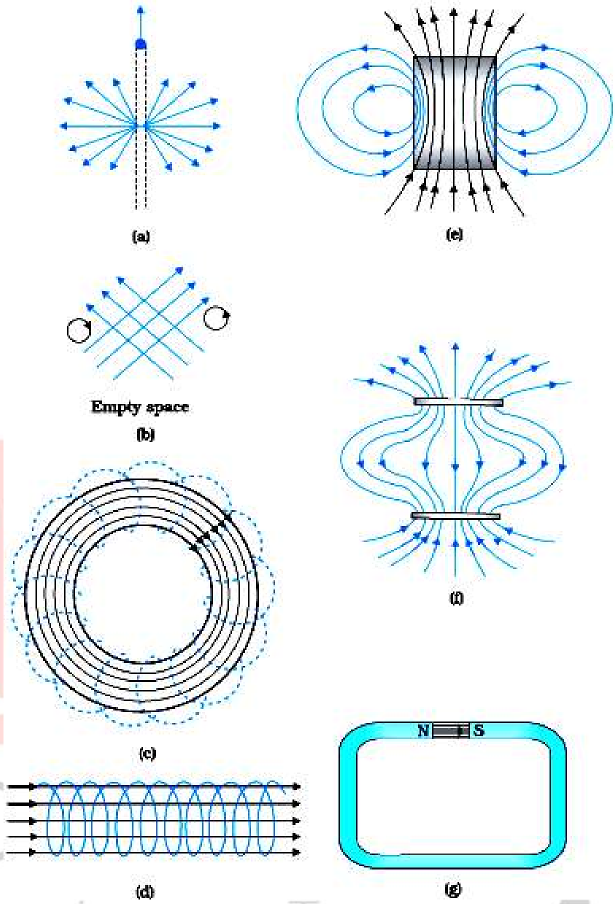
(b)If field lines were entirely confined between two ends of a straight solenoid, the flux through the cross-section at each end would be non-zero. But the flux of field $\vec{\mathbf{B}}$ through any closed surface must always be zero. For a toroid, this difficulty is absent because it has no 'ends'.

(c) No

(d)Yes

QUESTION

Many of the diagrams given in fig. (2) show magnetic field lines (thick lines in the figure) wrongly. Point out what is wrong with them. Some of them may describe electrostatic field lines correctly. Point out which ones.



[fig.(2)]

ANS-

(a)Wrong

(b)Wrong

(c)Right

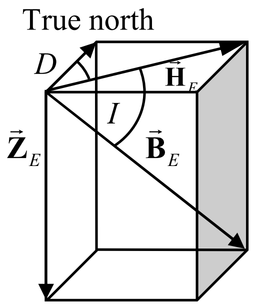
(d)Wrong

(e)Right

(f)Wrong

(g)Wrong

EARTH'S MAGNETIC FIELD



[fig.(1)]

The magnetic field of the earth is now thought to arise due to electrical currents produced by convective motion of metallic fluids (consisting mostly of molten iron and nickel) in the outer core of the earth. This is known as the dynamo effect.

The magnetic field lines of the earth resemble that of a (hypothetical) magnetic dipole located at the centre of the earth. The axis of the dipole does not coincide with the axis of rotation of the earth but is presently titled by approximately 11.3º with respect to the later. The location of the north magnetic pole is at a latitude of 79.74º N and a longitude of 71.8º W, a place somewhere in north Canada. The magnetic south pole is at 79.74º S, 108.22º E in the Antarctica.

If one looks at the magnetic field lines of the earth, one sees that unlike in the case of a bar magnet, the field lines go into the earth at the north magnetic pole (Nm) and come out from the south magnetic pole (Sm).

The north pole of a magnet was so named as it was the north-seeking pole. Thus, in reality, the north magnetic pole behaves like the south pole of a bar magnet inside the earth and vice versa.

The vertical plane containing the longitude circle and the axis of rotation of the earth is called the geographic meridian. In a similar way, one can define magnetic meridian of a place as the vertical plane which passes through the imaginary line joining the magnetic north and the south poles. A magnetic needle, which is free to swing horizontally, would then lie in the magnetic meridian and the north pole of the needle would point towards the magnetic north pole.

The magnetic meridian at a point makes an angle with the geographic meridian. This, then, is the angle between the true geographic north and the north shown by a compass needle. This angle is called the magnetic declination or simply declination.

The declination is greater at higher latitudes and smaller near the equator. The declination in India is small, it being $0°41'$ E at Delhi and $0°58'$ W at Mumbai.

the angle of dip (also known as inclination) is the angle that the total magnetic field $\vec{\mathbf{B}}_E$ of the earth makes with the surface of the earth.

To describe the magnetic field of the earth at a point on its surface, we need to specify three quantities, viz., the declination D, the angle of dip or the inclination I and the horizontal component of the earth's field H_E. These are known as the element of the earth's magnetic field.

So

$$\tan I = \frac{Z_E}{H_E}$$

PROBLEM

In the magnetic meridian of a certain place, the horizontal component of the earth's magnetic field is 0.26G and the dip angle is 60º. What is the magnetic field of the earth at this location? (NCERT)

SOL-

$$B_E = \frac{H_E}{\cos 60^\circ} = 0.52\text{G}$$

QUESTION

Answer the following questions regarding earth's magnetism:

(a) A vector needs three quantities for its specification. Name the three independent quantities conventionally used to specify the earth's magnetic field.

(b) The angle of dip at a location in southern India is about 18º. Would you expect a greater or smaller dip angle in Britain?

(c) If you made a map of magnetic field lines at Melbourne in Australia, would the lines seem to go into the ground or come out of the ground?

(d) In which direction would a compass free to move in the vertical plane point to, if located right on the geomagnetic north or south pole?

(e) The earth's field, it is claimed, roughly approximates the field due to a dipole of magnetic moment $8\times10^{22}\,\mathrm{J\cdot T^{-1}}$ located at its centre. Check the order of magnitude of this number in some way.

(f) Geologists claim that besides the main magnetic N-S poles, there are several local poles on the earth's surface oriented in different directions. How is such a thing possible at all? (NCERT)

MAGNETISATION AND MAGNETIC INTENSITY

We define the magnetisation $\vec{\mathbf{M}}$ of a sample to be equal to its net magnetic moment per unit volume:

$$\vec{\mathbf{M}} = \frac{\vec{\mathbf{m}}_{net}}{V}$$

$\vec{\mathbf{M}}$ is a vector with dimensions $L^{-1}A$ and is measured in a units of A/m.

Consider a long vacuum core solenoid of n turns per unit length and carrying a current I. The magnetic field in the interior of the solenoid is given by

$$B_0 = \mu_0 nI$$

If the interior of the solenoid is filled with a material with non-zero magnetisation, the field inside the solenoid will be greater than B_0. The net magnetic field in the interior of the solenoid may be expressed as

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_0 + \vec{\mathbf{B}}_m$$

where $\vec{\mathbf{B}}_m$ is the field contributed by the material core? It turns out that this additional field $\vec{\mathbf{B}}_m$ is proportional to the magnetisation $\vec{\mathbf{M}}$ of the material and is expressed as

$$\vec{\mathbf{B}}_m = \mu_0\vec{\mathbf{M}}$$

It is convenient to introduce another vector field $\vec{\mathbf{H}}$, called the magnetic intensity, which is defined by

$$\vec{\mathbf{H}} = \frac{\vec{\mathbf{B}}}{\mu_0} - \vec{\mathbf{M}}$$

It is seen that

$$\vec{\mathbf{M}} = \chi\vec{\mathbf{H}}$$

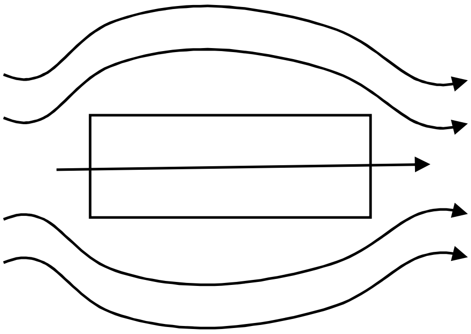
where, a dimensionless quantity χ, is appropriately called the magnetic susceptibility. It is a measure of how a magnetic material responds to an external field. So

$$\vec{\mathbf{B}} = \mu\vec{\mathbf{H}}$$

where,

$\mu = \mu_0\mu_r = \mu_0(1+\chi)$ and μ_r is the relative magnetic permeability of the substance.

PARA-, DIA- AND FERRO-MAGNETIC SUBSTANCES



$[fig.(1)]$

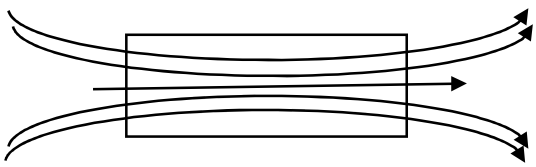
Diamagnetic substances are those which tend to move from stronger to the weaker part of the external magnetic field. $-1 \le \chi < 0$, $0 \le \mu_r < 1$, $\mu < \mu_0$,

Figure (1) shows a bar of diamagnetic material placed in an external magnetic field. The field lines are repelled or expelled and the field inside the material is reduced.

Diamagnetic substances are the ones in which resultant magnetic moment in an atom is zero. When a magnetic field is applied, those electrons having an orbital magnetic moment in the same direction slow down and those in the opposite direction speed up. This happens due to induced current in accordance with Lenz's. Thus, the substance develops a net magnetic moment in the direction opposite to that of the applied field and hence repulsion.

Superconductors are metals, cooled to very low temperatures which exhibits both perfect

conductivity and perfect diamagnetism. Here the field lines are completely expelled! $\chi = -1$ and $\mu_r = 0$. The phenomenon of perfect diamagnetism in superconductors is called the Meissner effect, after the name of its discoverer.



$[fig.(2)]$

Paramagnetic substances get weakly attracted to a magnet. The individual atoms (or ions or molecules) of a paramagnetic material possess a permanent magnetic dipole moment of their own. In the presence of an external field $\vec{\mathbf{B}}_0$, which is strong enough, and at low temperatures, the individual atomic dipole moment can be made to align and point in the same direction as $\vec{\mathbf{B}}$. $0 < \chi < \varepsilon$, $1 < \mu_r < 1 + \varepsilon$, $\mu > \mu_0$

Some paramagnetic materials are aluminium, sodium, calcium, oxygen (at STP) and copper chloride.

Magnetisation of a paramagnetic material is inversely proportional to the absolute temperature T,

$M = C\frac{B_0}{T}$ or equivalently $\chi = C\frac{\mu_0}{T}$.

This is known as Curie's law, after its discoverer Pieree Curie (1859-1906). The constant C is called Curie's constant.

Ferromagnetic substances are those which gets strongly magnetised when placed in an external magnetic field. They have a strong tendency to move from a region of a weak magnetic field to a strong magnetic field, i.e., they get strongly attracted to a magnet.

Ferromagnetic substances are those which gets strongly magnetised when placed in an external magnetic field. They have a strong tendency to move from a region of a weak magnetic field to a strong magnetic field, i.e., they get strongly attracted to a magnet. $\chi \gg 1$, $\mu_r \gg 1$, $\mu \gg \mu_0$ Each domain has a net magnetisation. When we apply an external magnetic field $\vec{\mathbf{B}}_0$, the domains orient themselves in the direction of $\vec{\mathbf{B}}_0$ and simultaneously the domain-oriented in the direction of $\vec{\mathbf{B}}_0$ grow in size. The susceptibility above the Curie temperature, i.e., in the paramagnetic phase is described by,

$$\chi = \frac{C}{T - T_0}(T > T_C).$$

PROBLEM

A domain in ferromagnetic iron is in the form of a cube of side length $1\mu m$. Estimate the number of iron atoms in the domain and the maximum possible dipole moment and magnetisation of the domain. The molecular mass of iron is 55 g/mole and its density is 7.9 g/cm3. Assume that each iron atom has a dipole moment of $9.27\times10^{-24}\,\mathrm{A\cdot m^2}$.

ELECTROMAGNETS AND FACTORS AFFECTING THEIR STRENGTHS, PERMANENT MAGNETS

Substances which at room temperature retain their ferromagnetic property for a long period of time are called permanent magnets. Permanent magnets can be made in a variety of ways. One can hold an iron rod in the north-south direction and hammer it repeatedly. An efficient way to make a permanent magnet is to place a ferromagnetic rod in a solenoid and pass current. The magnetic field of the solenoid magnetises the rod. The material should have high retentivity so that the magnet is strong and high coercivity so that the magnetisation is not erased by stray magnetic fields, temperature fluctuations or minor mechanical damage. Further, the material should have a high permeability. Steel is one-favoured choice. It has a slightly smaller retentivity than soft iron but this is outweighed by the much smaller coercivity of soft iron. Other suitable materials for permanent magnets are alnico, cobalt, steel and ticonal.

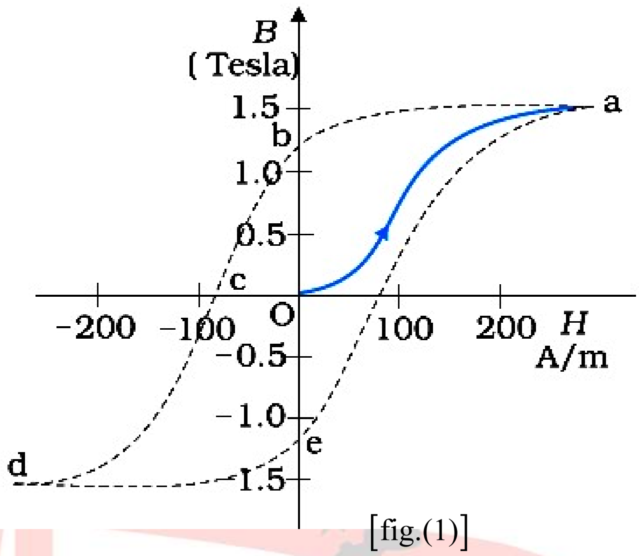
Core of electromagnets are made of ferromagnetic materials which have high permeability and low retentivity. Soft iron is a suitable material for electromagnets. On placing a soft iron rod in a solenoid and passing a current, we increase the magnetism of the solenoid by a thousand fold. When we switch off the solenoid current, the magnetism is effectively switched off since the soft iron core has a low retentivity.

In certain applications, the material goes through an ac cycle of magnetisation for a long period. This is the case in transformer cores and telephone diaphragms. The hysteresis curve of such materials must be narrow. The energy dissipated and the heating will consequently be small.

The material must have a high resistivity to lower eddy current losses. Electromagnets are used in electric bells, loudspeakers and telephone diaphragms. Giant electromagnets are used in cranes to lift machinery, and bulk quantities of iron and steel.

HYSTERESIS

The relation between B and H in ferromagnetic materials is complex. It is often not linear and it depends on the magnetic history of the sample. Figure (1) depicts the behaviour of the material as we take it through one cycle of magnetisation. Let the material be unmagnetised initially. We place it in a solenoid and increase the current through the solenoid. The magnetic field B in the material rises and saturates as depicted in the curve Oa. This behaviour represents the alignment and merger of domains until no further enhancement is possible. It is pointless to increase the current (and hence the magnetic intensity H) beyond this. Next, we decrease H and reduce it to zero. At $H = 0$, $B \neq 0$. This is represented by the curve ab. The value of B at $H = 0$ is called retentivity or remanence. In Fig. (1), $B_R \sim 1.2\text{T}$. The domains are not completely randomised even though the external driving field has been removed. Next, the current in the solenoid is reversed and slowly increased. Certain domains are flipped until the net field inside stands nullified. This is represented by the curve bc. The value of H at c is called coercivity. In Fig. (1) $H_c \sim 90\text{A/m}$. As the reversed current is increased in magnitude, we once again obtain saturation. The curve cd depicts this. The saturated magnetic field $B_s \sim 1.5\text{T}$. Next, the current is reduced (curve de) and reversed (curve ea). The cycle repeats itself. Note that the curve Oa does not retrace itself as H is reduced. For a given value of H, B is not unique but depends on the previous history of the sample. This phenomenon is called hysteresis. The word hysteresis means lagging (and not 'history')



[fig.(1)]