

Linear Inequation and its Properties

SUBJECT : MATHEMATICS

CHAPTER NUMBER: 06

CHAPTER NAME : LINEAR INEQUALITIES

CHANGING YOUR TOMORROW

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Learning Objectives:

- Students will be able to learn about the difference between equalities and inequalities.
- Students will be able to learn about different types of inequalities.
- Students will be able to learn how to solve linear inequalities in one variable.
- Students will be able to learn to find the graphical solution linear inequalities in two variables.
- Students will be able to learn to solve system of linear inequalities.
- Students will be able to implement application- oriented skills in their day -to -day life.

Introduction:

The system of real numbers can be arranged in such a way that each number is greater than or less than any other number. Such an arrangement is called the inequalities in real number system.

The symbols ' $>$ ', ' $<$ ', ' \geq ', ' \leq ' are called the signs of inequalities.

Inequalities:

Two real numbers or two algebraic expressions related by the symbol ' $>$ ', ' $<$ ', ' \geq ' or ' \leq ' form an inequality.

For example: $4 > 2$, $x \leq 5$, $2x + 3y > 6$.

Different Forms of Inequalities:

1. Numerical inequalities:

2. Literal inequalities:

3. Linear inequalities in one variable:

$ax + b > 0, ax + b < 0, ax + b \leq 0, ax + b \geq 0, ax + b \leq c, ax + b \leq c$, where a, b, c are real numbers and $a \neq 0$.

4. Linear inequalities in two variables:

$ax + by > c, ax + by < c, ax + by \leq c, ax + by \geq c$, where a, b, c are real numbers and $a \neq 0, b \neq 0$.

5. Strict inequalities: The inequalities involving the symbol ' $<$ ' or ' $>$ ' are called strict inequalities.

6. Slack inequalities: The inequalities involving the symbol ' \geq ' or ' \leq ' are called slack inequalities.

7. Unconditional inequalities:

For example: $x^2 + 5 > 2x + 1$ is true for all real values of x .

8. Conditional inequalities:

For example: $3x + 2 > 14$ is true only for those values of x which are greater than 4.

Rules of Inequalities:

1. If $a > b$ and c is any number, then $a + c > b + c$ and $a - c > b - c$.
2. If $a > b$ and $c > 0$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$.
3. If $a > b$ and $c < 0$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$
4. $a - c > b \Rightarrow a > b + c$ or $-c > b - a$
5. If $a > b$, then $-a < -b$.
6. If $a > b$ then $a^n > b^n$ and $a^{-n} < b^{-n}$, $n \in N$.

Solutions of an inequation:

The value(s) of the variable(s) which makes the inequality a true statement is called its solution.

Solving an Inequation:

It is the process of obtaining all possible solutions of an inequation.

Solution Set:

The set of all solutions of an inequality is called the solution set.

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Algebraic solutions of linear inequation in one variable and graphical representation

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Solving Linear Inequation in One Variable:

Solving inequations of the form $ax > b, ax \geq b, ax < b, ax \leq b$ ($a \neq 0$)

Example: Solve $20x < 105$ when

(i) x is a natural number (ii) x is an integer.

Example: Solve $-30x \leq 200$, when

(i) x is a natural number (ii) x is an integer (iii) x is a real number.

Example: Solve the following linear inequations:

$$(i) 2x - 4 \leq 0.$$

$$(ii) 7x + 9 > 30.$$

$$(iii) -3x + 12 \geq 0.$$

Solving inequations of the form

$ax + b > cx + d$ or $ax + b < cx + d$ or $ax + b \geq cx + d$ or $ax + b \leq cx + d$

Example: Solve: $5x - 3 < 3x + 1$ when

- (i) x is a real number
- (ii) x is integer
- (iii) x is a natural number.

Example: Solve the following inequations:

$$(i) 3x + 17 \leq 2(1 - x)$$

$$(ii) 2(2x + 3) - 10 \leq 6(x - 2)$$

Example: Solve the following inequalities:

$$(i) \frac{2x-3}{4} + 9 \geq 3 + \frac{4x}{3}$$

$$(ii) \frac{5x-2}{3} - \frac{7x-3}{5} > \frac{x}{4}$$

$$(iii) \frac{1}{2} \left(\frac{3}{5}x + 4 \right) \geq \frac{1}{3}(x - 6)$$

$$(iv) \frac{1}{x-2} < 0$$

$$(v) \frac{3(x-2)}{5} \geq \frac{5(2-x)}{3}$$

Solving Inequation of the form $\frac{ax+b}{cx+d} > k$ or $\frac{ax+b}{cx+d} \geq k$ or $\frac{ax+b}{cx+d} < k$ or $\frac{ax+b}{cx+d} \leq k$

Example: Solve the following inequations.

$$(i) \frac{2x+4}{x-1} \geq 5$$

$$(ii) \frac{x+3}{x-2} \leq 2$$

Solving inequations with a modulus sign

For this, we use the inequalities:

- For $a > 0$, $|x| \leq a \Leftrightarrow -a \leq x \leq a$
- $|x| \geq a \Leftrightarrow x \geq a \text{ or } x \leq -a$

Example: Solve $|5x + 3| < 4$.

Example: Express the following subset of R in the interval form: $\{x: |5x - 3| > 12\}$.

Example: Solve the following inequality and represent the solution set on the number line.

$$(i) x + \frac{x}{2} + \frac{x}{3} < 11$$

$$(ii) \frac{1}{2} \left(\frac{3}{5} x + 4 \right) \geq \frac{1}{3} (x - 6)$$

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Practical Problems based on Linear Inequalities

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Introduction:

In this section, we shall utilize the knowledge of solving linear inequations in one variable in solving different problems from various fields such as science, engineering , economics etc.

Example: Find all pairs consecutive odd positive integers, both of which are smaller than 18, such that their sum is more than 20.

Example: Find all pairs of consecutive even positive integers, both of which are larger than 8, such that their sum is less than 25.

Example: The cost and revenue functions of a product are given by $C(x) = 2x + 400$ and $R(x) = 6x + 20$ respectively, where x is the number of items produced by the manufacturer. How many items the manufacturer must sell to realize some profit?

Example: IQ of a person is given by the formula: $IQ = \frac{MA}{CA} \times 100$, where MA is mental age and CA is chronological age. If $80 \leq IQ \leq 140$ for a group of 12 year children, find the range of their mental age.

Example: In the first four papers each of 100 marks, Rishi got 95, 72, 73, 83 marks. If he wants an average of greater than or equal to 75 marks and less than 80 marks, find the range of marks he should score in the fifth paper.

Example: A manufacturer has 600 litres of a 12% solution of acid. How many litres of a 30% acid solution must be added to it so that acid content in the resulting mixture will be more than 15% but less than 18%.

Example: A man wants to cut three lengths from a single piece of board of length 91 cm. The second length is to be 3 cm longer than the shortest and third length is to be twice as long as the shortest. What are the possible lengths for the shortest board if third piece is to be at least 5 cm longer than the second?

Solution of system of Linear inequations in one variable:

The solution set of a system of linear inequations in one variable is the intersection of the solution sets of the linear inequations in the given system.

Example: Solve the following system of linear inequations:

$$3x - 6 \geq 0, 4x - 10 \leq 6.$$

Example: Solve: $-5 \leq \frac{2-3x}{4} \leq 9.$

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